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## OPTIMIZATION OF PILE DESIGN FOR OFFSHORE WIND TURBINE JACKET FOUNDATIONS

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### ABSTRACT

The aim of this study is to use numerical methods of structural design optimization to design piles for offshore wind turbine jacket foundations. Pile mass is minimized with constraints on axial and lateral capacity. Results indicate that accurate knowledge about soil characteristics can translate into significant cost reductions.

### NOMENCLATURE

$d$	=	External diameter of the pile (m)
$d_i$	=	Internal diameter of the pile (m)
$l$	=	Length of the pile (m)
$t$	=	Thickness of pile wall (m)

### INTRODUCTION

Foundation and substructure for offshore wind turbines can amount to more than 20 % of the capital expenditure in a project. Pile-anchoring a jacket foundation to the seabed requires to first perform a geological survey to map the soil characteristics and then design a pile with sufficient capacity to carry the loads from the substructure.

Structural optimization [1] is the science of achieving the optimal load carrying structure. Given an objective such as compliance or cost, and some constraints such as limitations on the maximum stress or displacement, the optimal structure is defined as the one that minimizes the objective function while satisfying the constraints.

A pile foundation is a slender hollow cylinder that is inserted into the soil. Anchoring by driven piles has been used extensively in the oil and gas industry, and in the last decade also for offshore wind turbines using either monopiles or piled jacket foundations. Pile foundation design is considered in this study according to the current state of practice for offshore foundations, see eg. [2], [3]. Axial and lateral capacities are calculated based on shaft friction and end bearing resistance and lateral resistance respectively. The obtained ultimate loads formulate the design basis.

### MODEL

An analysis and optimization tool for jacket design, JADOP, has been developed at DTU Wind Energy. In this study JADOP is expanded to include pile design. JADOP is a finite element package developed in Matlab, with specific features for design optimization of wind turbine support structures. The tower and substructure are described by Timoshenko beam elements, and the load set is applied by nodal forces at the tower top (rotor loads), throughout the structure (self-weight), and throughout the submerged part of the jacket (wave loads).

The piles are assumed to be slender hollow cylinders with constant diameter and thickness. For such piles, the Randolph formulation [4] provides an analytic relationship between forces and displacements at the pile head:

$$\begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} K_{su} & 0 & 0 & 0 & -K_{s\theta} & 0 \\ 0 & K_{su} & 0 & K_{s\theta} & 0 & 0 \\ 0 & 0 & K_v & 0 & 0 & 0 \\ 0 & K_{mu} & 0 & K_{m\theta} & 0 & 0 \\ -K_{mu} & 0 & 0 & 0 & K_{m\theta} & 0 \\ 0 & 0 & 0 & 0 & 0 & K_T \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ u_z \\ \theta_x \\ \theta_y \\ \theta_z \end{bmatrix} \quad (1)$$

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All the entries in the pile stiffness matrix are continuous differentiable functions of soil properties and pile design variables:

$$K_{su} = 6.29426 G_s \left( \frac{E_{eq}}{G_s} \right)^{\frac{1}{7}} \frac{d}{2}, \quad K_{s\theta} = K_{mu} = 2.12431 \left( \frac{E_{eq}}{G_s} \right)^{\frac{3}{7}} \left( \frac{d}{2} \right)^2, \quad K_{mc} = 1.96696 \left( \frac{E_{eq}}{G_s} \right)^{\frac{5}{7}} \left( \frac{d}{2} \right)^3 \quad (2)$$

$$K_T = G_p d^3 \left( \frac{2}{3} + \frac{\pi l}{d} \right), \quad K_v = \frac{2 G_p d}{1 - \nu} \left( 1 + \frac{(1 - \nu) \pi l \tanh \mu l}{\zeta d \mu l} \right) \quad (3)$$

$$E_{eq} = \frac{E_s I_s + E_p I_p}{I_s + I_p}, \quad I_p = \frac{\pi(d^2 - d_i^2)}{64}, \quad (\mu l)^2 = \frac{2 G_p}{\zeta E_p} \left( \frac{2l}{d} \right)^2, \quad \zeta = \ln \frac{5l(1 - \nu)}{d} \quad (4)$$

where  $E_s$ ,  $G_s$ ,  $I_s$ ,  $E_p$ ,  $G_p$ , and  $I_p$  are the Young's modulus, shear modulus, and inertia of soil and pile, respectively.  $\nu$  is the Poisson ratio of the pile material.

Axial pile capacity can be divided into friction capacity,  $Q_f$ , and end bearing capacity  $Q_b$ . The pile capacity in compression and tension are defined as  $Q_c = Q_f + Q_b$  and  $Q_t = Q_f$ , respectively.  $Q_f$  and  $Q_b$  are given for clay, see equations (5-6), and for sand, see equations (7-8), where  $b_\phi$ ,  $f_{max,\phi}$ ,  $N_{q,\phi}$ , and  $q_{b,max,\phi}$  are defined with respect to friction angle  $\phi$  in Table 1 [3].

$$Q_f^{clay}(d, l) = \int_{z=0}^{z_1} \pi d \frac{1}{2} \left( \frac{s_u}{\gamma' z} \right)^{-\frac{1}{2}} s_u dz + \int_{z_1}^L \pi d \frac{1}{2} \left( \frac{s_u}{\gamma' z} \right)^{-\frac{1}{4}} s_u dz, \quad z_1 = \frac{s_u}{\gamma'} \quad (5)$$

$$Q_b^{clay}(d, t, l) = \min \left( 9 s_u \frac{\pi d^2}{4}, \quad 9 s_u \frac{\pi(d^2 - d_i^2)}{4} + Q_f(d_i) \right) \quad (6)$$

$$Q_f^{sand}(d, l) = \int_{z=0}^{z_1} \pi d b_\phi \gamma' z dz + \int_{z_1}^L \pi d f_{max,\phi} dz, \quad z_1 = \frac{f_{max,\phi}}{b_\phi \gamma'} \quad (7)$$

$$Q_b^{sand}(d, t, l) = \min \left( \min(N_{q,\phi} \gamma' L, q_{b,max,\phi}) \frac{\pi d^2}{4}, \quad \min(N_{q,\phi} \gamma' L, q_{b,max,\phi}) \frac{\pi d_i^2}{4} + Q_f(d_i, l) \right) \quad (8)$$

Lateral resistance is assumed to be the yield bending moment of the pile cross section:

$$Q_M(d, t) = \frac{2 \sigma_y I_p(d, t)}{D} \quad (9)$$

where  $\sigma_y = 350$  MPa is the yield stress of the pile. Equivalent lateral load is computed by Broms theory [5]:

$$M_{eq}^{clay}(d, t, l) = \frac{0.5}{9 s_u d} F^2 + 1.5 d F + M, \quad M_{eq}^{sand}(d, t, l) = \frac{0.544}{(d K_p \gamma')^{0.5}} F^{1.5} + M, \quad K_p = \tan^2(45 + \frac{\phi}{2}) \quad (10)$$

The design basis is now defined as  $-Q_t \leq F_z \leq Q_c$  and  $-Q_M \leq M_{eq} \leq Q_M$  for each of the four piles, and lateral capacities in both x- and y-directions. Since this design basis is only valid for slender piles, we constrain the length to be at least 10 times the diameter. For driven piles, API also recommends the pile thickness  $t \geq 0.0063 + \frac{D}{100}$  (m) [3].

## OPTIMIZATION PROBLEM

Let the variables be defined as diameter, thickness, and length of a pile, and let the four piles be identical. The variables are then bounded above and below to avoid unrealistic designs. The objective function is pile mass, since this is assumed to be the main cost driver. The constraints are the design basis for the piles.

$$\begin{aligned} & \underset{\mathbf{x}=(d,t,l) \in \mathbb{R}^3}{\text{minimize}} && c(\mathbf{x}) = \rho \pi (d^2 - (d - 2t)^2) l \\ & \text{subject to} && -F_{z,i}(\mathbf{x}) - Q_t(\mathbf{x}) \leq 0, \quad i = 1, \dots, 4 \\ & && F_{z,i}(\mathbf{x}) - Q_c(\mathbf{x}) \leq 0, \quad i = 1, \dots, 4 \\ & && -M_{eq}(\mathbf{x}) - Q_M(\mathbf{x}) \leq 0, \quad i = 1, \dots, 4 \\ & && M_{eq}(\mathbf{x}) + Q_M(\mathbf{x}) \leq 0, \quad i = 1, \dots, 4 \\ & && d - \frac{l}{10} \leq 0 \end{aligned} \quad (12)$$

$$0.0063 + \frac{d}{100} - t \leq 0$$

$$\underline{x} \leq x \leq \bar{x}$$

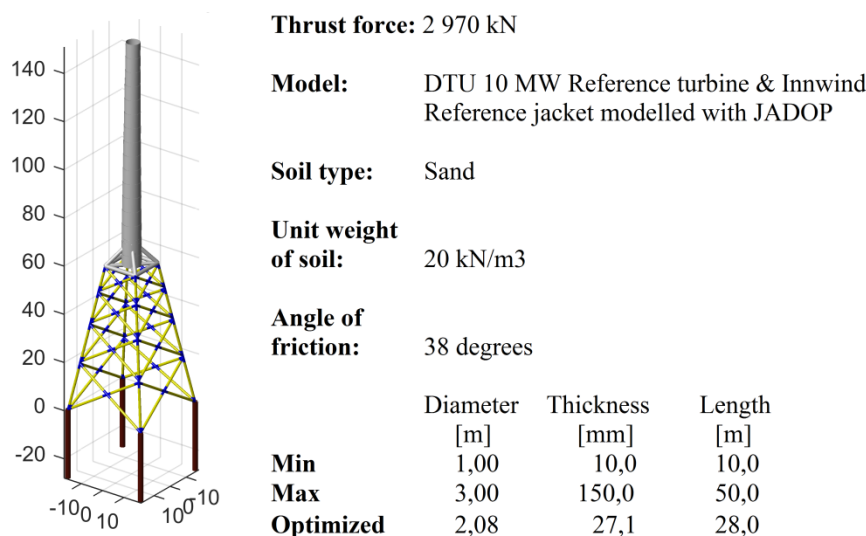
Note that both the forces and the capacities are functions of the design variables. In the capacities there are some non-smooth functions, but this can in all cases be taken care of by splitting the non-smooth constraint into multiple constraints that are smooth. The sensitivities are computed analytically for all functions, but for brevity they are not written out here. The problem is formulated as a simultaneous analysis and design (SAND) problem. This means that i) the finite element system does not have to be solved in every iteration, but is imposed as a constraint, and ii) any solution that has not converged is in general not a solution at all. In the numerical examples shown here, all optimization runs have converged, so this is not a problem.

## NUMERICAL EXAMPLES

Three numerical examples are presented in this study to demonstrate the feasibility of pile design optimization and how it can be used to gain further insight about the engineering process:

1. The optimization problem in equations (12) is solved once, see Figure 1.
2. The optimization problem in equations (12) is solved a number of times with incremental change in the soil properties of clay.
3. The optimization problem in equations (12) is solved a number of times with incremental change in the soil properties of sand.

The structure above the piles is modelled to replicate the Innwind reference jacket [6], and is not modified in the optimization. Note also that the initial pile design is an average of the variable bounds, and does not represent any reference design. JADOP computes analysis and sensitivities, and the optimization problem is solved in IPOPT [7].



**Figure 1.** Overview of numerical example #1. The structural model and variable bounds are also used in numerical examples #2 and #3.

**Table 1.** Soil properties of sand which depend on the friction angle of the soil. See equations (2-4) for sand capacities.

$\varphi$ [°]	$b_{\varphi}$ [-]	$f_{max,\varphi}$ [kPa]	$N_{q,\varphi}$ [-]	$q_{b,max,\varphi}$ [MPa]
33-35	0.29	67	12	3
35-37	0.37	81	20	5
37-40	0.46	96	40	10
40-42	0.56	115	50	12

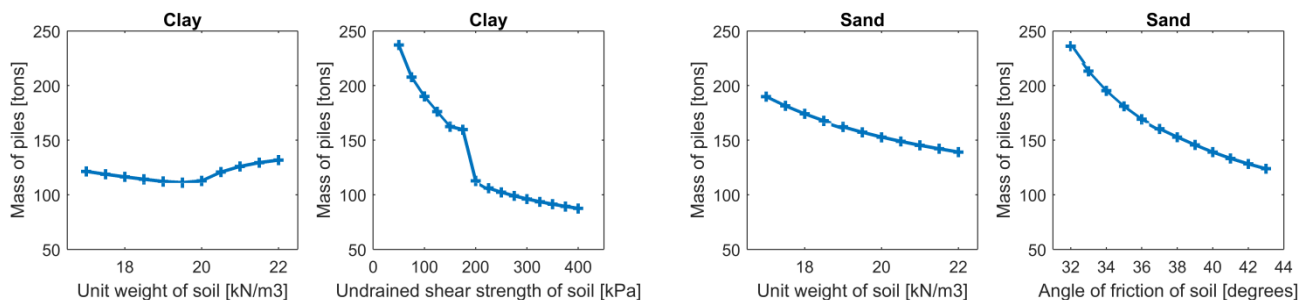
## RESULTS AND DISCUSSION

1. The optimized mass was 153 tons for the four piles in total, and the diameter, thickness, and length was 2080 mm, 27.1 mm, and 28.0 m respectively (Figure 1). The optimization converged after 20 iterations in less than 10 seconds. This example shows that the optimization problem (12) can be used to optimize the pile design for a given structure

when the geotechnical data is known. Without any preconceptions about the design, except that it should be a slender hollow cylinder, the solution to (12) generate a pile design that has low mass and satisfies the design basis.

2. Numerical example #2 (Figure 2, clay) shows how sensitive the pile design is to the weight and shear strength of the soil. All data points represent a pile design that has been optimized in the same way as numerical example #1. Pile design in clay is very sensitive to undrained shear strength, and the higher the strength, the lower the pile mass.
3. Numerical example #3 (Figure 2, sand) is similar to numerical example #2. Pile design in a sandy soil is sensitive both to the weight of the soil, and to the angle of friction. For average soil conditions it appears that sandy soil requires heavier piles.

The numerical examples show that changes in soil properties can change the required pile mass with a factor of as much as 2 or 3. For a large offshore wind farm site with varying soil properties, there can therefore be a large cost reduction potential if one maps the geotechnical data all over the site. Then one can optimize the pile design specifically for each turbine, and avoid expensive conservative designs. In such situation, the advantage of an automatic pile optimization methodology as presented here is particularly useful.



**Figure 2.** Results of numerical example #2 (clay) and #3 (sand). See Table 1 for parameters depending on the friction angle in sandy soil. The non-smooth behavior of the pile mass with respect to undrained shear strength of clay is assumed to be due to a local minimum in the optimization.

## CONCLUSION

It is demonstrated that preliminary pile design can be automated using structural optimization. We observed that in clay, the pile design is highly sensitive to the shear strength of the soil. For piles in sand, we observed that both unit weight and angle of friction has a high influence on the total mass of the piles.

The pile mass can change with a factor of 2 or 3 when the soil parameters change. This implies that the geotechnical data should be mapped for each wind turbine in the offshore wind turbine site. Furthermore, if the piles are designed specifically for each turbine, one can obtain significant cost reductions compared to the case where the worst condition is used for all pile designs.

The main limitation of this study is that the design basis assumes only one soil layer, while multiple layers is often the case in reality.

## ACKNOWLEDGEMENTS

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